Short Papers

Composite Hole Conditions on Complex Permittivity Measurements Using Microwave Cavity Perturbation Techniques

SHIHE LI, STUDENT MEMBER, IEEE, AND RENATO G. BOSISIO, MEMBER, IEEE

***Abstract*— This paper examines the edge effects of introducing a dielectric test cylinder with a test material into a cavity via a metallic support tube extending outside the cavity. A first-order perturbation theory is used for this metallic hole containing two different concentric dielectric materials. The Galerkin method is used to determine the amplitudes of numerous evanescent modes which exist in such composite hole geometries. Comparisons are made with the effects produced by a simple hole in which a single dielectric is postulated inside the metallic support tube. The effects of the composite hole on the measurement of the dielectric properties of materials are given.**

I. Introduction

In the field of dielectric measurements by means of the well-known cavity perturbation method, the effects of the sample insertion hole are probably the major source of measurement error. Estin and Bussey had published in 1960 a paper to discuss the problem of a simple hole in both theoretical and experimental ways [1]. In recent years, by means of the same procedure—simple perturbation theory—the effects of a simple hole on measurement of ϵ' for both TM_{010} and TM_{011} cavities were given by Meyer [2]. In the above two references only the main cylindrical TM_{01} mode in the hole region was taken into account. Furthermore, even for this single mode treatment the amplitude was chosen rather arbitrarily. In addition, Thomassen [3] also proposed another revised dispersion equation to describe this effect based on an approximate calculation of end capacitance of a simple plasma column. These results were checked by Gregory [4] via a series of measurements for some well-known materials in different TM_{010} cavities. From these results one can find that Thomassen's equations give a correction factor which is too big.

In this paper, the hole formed by an external metallic support tube, joined to the cavity, is considered to contain a dielectric test cylinder for introducing a sample material with varying dielectric properties as shown in Fig. 1. A large number of higher order modes are considered in this composite hole condition as compared to the single mode treatment of the simple hole [1], [2] containing only one dielectric. The results show that higher order modes in the composite hole converge rather slowly and will obviously contribute to the frequency perturbation. In addition, it is shown that the composite hole perturbs the cavity Q factor and a correction term for dielectric losses is also derived with the same theory. All these effects lead to the fact that measured ϵ' and ϵ'' are always found to be respectively smaller and higher than the value measured with no hole of the sample material. By

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The authors are with Laboratoire d'Hyperfrequences, Department De Genie Electrique, Ecole Polytechnique, P.O Box 6079, Station A, Montreal, Quebec, Canada H3C 3A7.

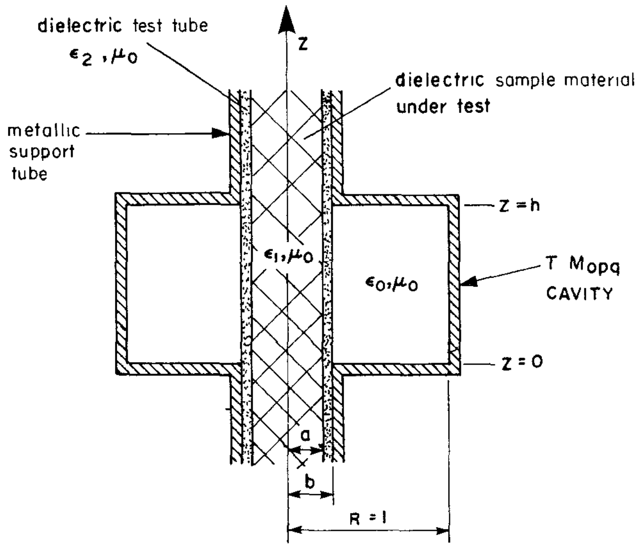

Fig. 1. Configuration of TM_{0pq} cavity with a concentric dielectric sample and its composite insertion hole conditions.

the use of the correction terms developed for the composite hole condition, the calculated error of the dielectric constant measurement is greatly reduced and much more accurate results can be obtained [5].

II. The Field Distribution in the Composite Hole Region

As shown in Fig. 1, we assume the lengths of dielectric sample, dielectric tube (to introduce the sample), and the metallic support tube (made with an ideal conductor) are infinite. This assumption is always satisfied when the tube's inner diameter is much smaller than the resonant wavelength in the dielectric, or when the following equation is satisfied:

$$p^2 b^2 \epsilon' \ll 1 \tag{1}$$

where

- p the radial index of the TM_{0pq} mode;
- b the tube's inner radius normalized by the radius of the cavity; and
- ϵ' the real part of the largest of the two dielectric constants: test tube and sample material.

Under this assumption and considering the symmetry of configuration, only cylindrical TM_{0n} modes can exist in the sample insertion tube; all these modes evanesce rapidly. The fields in the sample are given by $(0 \leq r \leq a)$

$$E_{z1} = \sum_{n=1}^{\infty} \tau_n A_n J_0(k_{rn} r) e^{-k_n z'} \tag{2.1}$$

$$E_{r1} = \sum_{n=1}^{\infty} \tau_n A_n J_1(k_{rn} r) e^{-k_n z'} \tag{2.2}$$

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and

$$H_{\phi 1}=j\omega\epsilon_0\epsilon_1 b\sum_{n=1}^{\infty}A_nJ_1(k_{rn}r)e^{-k_nz'} \tag{2.3}$$

where $k_{rn}^2-k_n^2=\epsilon_1k_0^2$, k_0 is the free-space wavenumber.

Because the attenuation constant k_n is always much larger than $\epsilon_1k_0^2$, we can approximately take

$$k_{rn}=k_n=\frac{\tau_n}{b}$$

with an error less than 1 percent when (1) is satisfied. By the same way, the fields in the dielectric tube are $(a\leqslant r\leqslant b)$

$$E_{z_2}=\sum_{n=1}^{\infty}\tau_nB_nZ_0(k_nr)e^{-k_nz'} \tag{3.1}$$

$$E_{r_2}=\sum_{n=1}^{\infty}\tau_nB_nZ_1(k_nr)e^{-k_nz'} \tag{3.2}$$

$$H_{\phi 2}=j\omega\epsilon_0\epsilon_2 b\sum_{n=1}^{\infty}B_nZ_1(k_nr)e^{-k_nz'} \tag{3.3}$$

where

$$Z_m(k_nr)=J_m(k_nr)Y_0(k_nb)-Y_m(k_nr)J_0(k_nb)$$

and

$$z'=\begin{cases}z-h, & \text{for upper hole}\\ -z, & \text{for bottom hole}.\end{cases}$$

According to the boundary condition at $r=a$, we can obtain the following relationship:

$$B_n=\frac{J_0(k_na)}{Z_0(k_na)}A_n$$

and

$$\epsilon_1J_1(k_na)Z_0(k_na)-\epsilon_2J_0(k_na)Z_1(k_na)=0. \tag{4}$$

From the characteristic equation (4), the mode series in the small sample test tube can be obtained. Fro the case $\epsilon_1=\epsilon_2$, we have

$$k_n=x_{0n}/b \tag{5}$$

where x_{0n} is the nth root of $J_0(x)=0$. This is the solution for the simplest configuration [1], [2].

Another problem is to find the amplitude spectrum A_n inside the tube. Because of the test tube perturbation, the RF field's distribution in the vicinity of interface $z'=0$ is destroyed and an E_r component appears. As an approximation, we assume that the H_ϕ component distribution in the cavity isn't perturbed by the hole. This leads

$$H_{\phi\ \text{tube}}=H_{\phi\ \text{unperturbed cavity}} \tag{6}$$

at $z'=0$ and $r\leqslant b$.

Substituting (2) and (3) into the above equation, and taking (1) into account, one can obtain

$$\sum_{n=1}^{\infty}A_nJ_1(\tau_nr/b)=\frac{r}{2b},\qquad 0\leqslant r\leqslant a \tag{7.1}$$

and

$$\sum_{n=1}^{\infty}\frac{A_nJ_0(\tau_na/b)}{Z_0(\tau_na/b)}Z_1(\tau_nr/b)=\frac{r}{2b}\left[1+\frac{(\epsilon_1'-\epsilon_2')a^2}{\epsilon_2'r^2}\right],\qquad a\leqslant r\leqslant b \tag{7.2}$$

where $\tau_n=k_nb$, and the small arguments of Bessel function have been used as an approximation.

TABLE I

AMPLITUDES A_n OF THE FIRST 20 MODES (TM_{01} THROUGH TM_{020}) FOR COMPOSITE HOLE ($\epsilon_2=4.75$, $a/b=0.7$)

N	$\epsilon=1$	$\epsilon=3$	$\epsilon=4.75$	$\epsilon=9$	$\epsilon=27$	$\epsilon=81$
1	0.785	0.691	0.666	0.670	0.816	1.211
2	−0.461	−0.272	−0.193	−0.105	−0.015	0.022
3	0.185	0.126	0.098	0.066	0.025	0.002
4	−0.056	−0.065	−0.062	−0.056	−0.041	−0.027
5	0.062	0.049	0.043	0.038	0.031	0.027
6	−0.068	−0.040	−0.033	−0.029	−0.024	−0.022
7	0.062	0.034	0.026	0.019	0.009	0.003
8	−0.087	−0.036	−0.021	−0.008	0.008	0.015
9	0.058	0.029	0.017	0.007	−0.007	−0.013
10	0.004	−0.016	−0.015	−0.013	−0.007	0.003
11	−0.015	0.010	0.013	0.016	0.017	0.016
12	−0.013	−0.013	−0.011	−0.013	−0.014	−0.014
13	0.043	0.016	0.010	0.009	0.008	0.009
14	−0.055	−0.019	−0.009	−0.004	0.001	0.003
15	0.046	0.018	0.008	0.001	−0.007	−0.010
16	−0.002	−0.011	−0.007	−0.005	0.002	0.007
17	−0.029	0.004	0.007	0.010	0.007	0.003
18	0.018	−0.005	−0.006	−0.010	−0.010	−0.010
19	0.022	0.010	0.006	0.007	0.007	0.008
20	−0.050	−0.014	−0.005	−0.003	−0.001	−0.001

It is obvious that the distribution of mode amplitudes in the metallic support tube does not depend on the resonant mode in the cavity and the radius of the tube b when (1) is satisfied and the first-order perturbation theory is used. To solve the above boundary problem, the Galerkin method [6] is useful. By taking N modes in the tube to match this tangential magnetic field at N uniformly distributed points, (7) then becomes a linear equation of order N. As an example, some typical solutions of A_n are listed in Table I, when a pyrex sampling tube ($\epsilon_2'=4.75$) is used to introduce a dielectric sample and N equals 40. When $N>20$, the amplitudes of the lower order modes, which are mainly contributing to storage and loss of RF energy, are effected only a little. From the results listed in Table I, one can find that the higher the ϵ_1 value, the faster the higher modes converge in the composed test tube case, and stronger effects on the ϵ' measurement will appear. When $\epsilon_1=\epsilon_2$, in the case of a single dielectric cylinder, the distribution of A_n will not depend on ϵ, and this conclusion can be obtained from (5) directly.

III. The Effects on Precise Measurement of ϵ^*

According to the adiabatic invariance theorem, the resonant frequency pulling due to a perturbation by the sample insertion tube will be equal to the variation of stored energy in the cavity. By use of the same procedure suggested by Estin and Bussey [1], and taking the results listed in Table I, this relative resonant frequency pulling for a TM_{0pq} cavity resonance is given by

$$\left(\frac{\delta f}{f_0}\right)_{pq}=\frac{a^2bx_{0p}^2K}{2\Delta_qhJ_1^2(x_{0p})\left(x_{0p}^2+q^2\pi^2/h^2\right)} \tag{8}$$

where

$$K=\sum_{n=1}^{\infty}\tau_nA_n^2J_0^2(\tau_na/b)\cdot\left\{\frac{\epsilon_2b^2Z_1^2(\tau_n)}{a^2Z_0^2(\tau_na/b)}+(\epsilon_1-\epsilon_2)\left[1-\frac{\epsilon_1J_1^2(\tau_na/b)}{\epsilon_2J_0^2(\tau_na/b)}\right]\right\} \tag{9}$$

and

$$\Delta_q = \begin{cases} 1, & \text{for } q=0 \\ 1/2, & \text{for } q \neq 0. \end{cases} \tag{10}$$

On the other hand, the existence of RF fields in the metallic support tube will produce an additional loss for lossy dielectric samples. It is found that

$$\delta\left(\frac{1}{Q_s}\right) = \frac{\int_{\text{tubes}} \epsilon_1'' E_{\text{tube}}^2 \, dv}{\int_{\text{cavity}} \epsilon_1'' E_{\text{sample}}^2 \, dv} \cdot \frac{1}{Q_s}$$

$$= \frac{b}{\Delta_q h Q_s} \sum_{n=1}^{\infty} \tau_n A_n^2 J_0^2 \left(1 + \frac{J_1^2}{J_0^2} - \frac{bJ_1}{a\tau_n J_0}\right) \tag{11}$$

where all arguments of Bessel function J_0 and J_1 are $\tau_n a/b$, and $1/Q_s$ is the loss of sample in the cavity. It is obvious that the measured variation in losses in cavity $\delta(1/Q_L)$ will not be equal to $1/Q_s$ (the dielectric loss of the sample).

In addition to the effects of cavity wall and external circuits (via coupling elements) already given [5], the sample loss $(1/Q_s)$ can be related to the loaded Q factor of cavity (Q_L) perturbed by tube only as follows:

$$\frac{1}{Q_s} = \left(1 - \frac{bD}{\Delta_q h}\right) \delta\left(\frac{1}{Q_L}\right) \tag{12}$$

where

$$D = \sum_{n=1}^{\infty} \tau_n A_n^2 J_0^2(\tau_n a/b) \left[1 + \frac{J_1^2(\tau_n a/b)}{J_0^2(\tau_n a/b)} - \frac{bJ_1(\tau_n a/b)}{a\tau_n J_0(\tau_n a/b)}\right].$$

By use of the simple perturbation theory, it is easily shown that

$$\frac{\delta(\epsilon' - 1)}{\epsilon' - 1} = \frac{bC}{\Delta_q h} \tag{13}$$

$$\frac{\delta \epsilon''}{\epsilon''} = -\frac{bD}{\Delta_q h} \tag{14}$$

where

$$C = \frac{K(\epsilon_1) - K(1)}{\epsilon_1' - 1}. \tag{15}$$

$K(\epsilon_1)$ and $K(1)$ are the values of K when $\epsilon_1 = \epsilon_1$ and $\epsilon_1 = 1$, respectively.

It must be noted that only the effects of one composite hole (upper or lower hole) is taken into account in the above calculation. The total effect in actual practice will be double the values indicated by (13) and (14).

IV. Results and Discussion

Numerical results of coefficients C and D in (13) and (14) are shown in Fig. 2 and Fig. 3, respectively. A discussion for the present theory can be made as follows.

1) The correction coefficient C (Fig. 2) will converge to one point $C = 0.323$ for any value of ϵ_1' when $a/b = 1$. This is also the same solution for the simplest configuration treated in the literature [1], [2], but with a much larger value (in the literature,

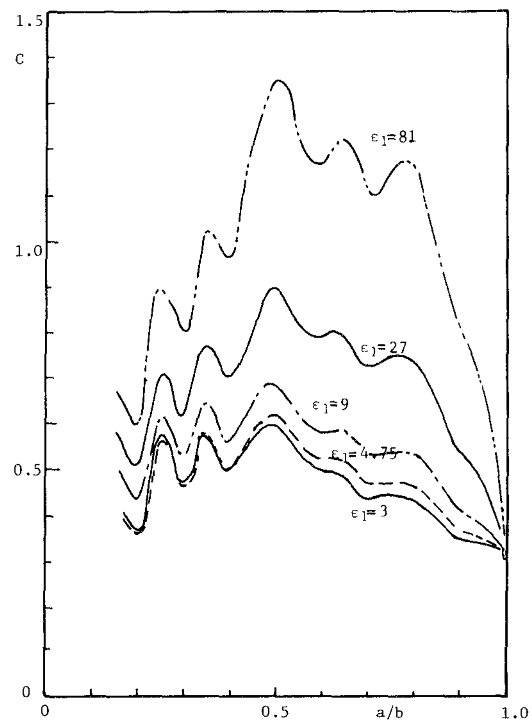

Fig. 2. The correction coefficient C versus a/b at different values of ϵ_1' for $\epsilon_2 = 4.75$.

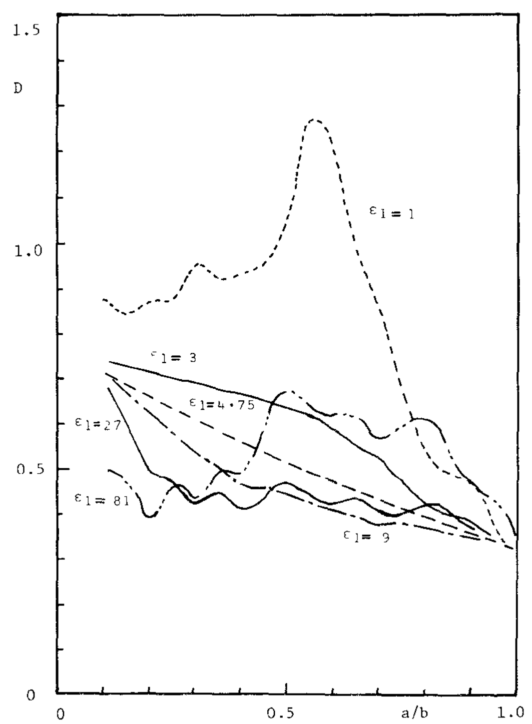

Fig. 3. The correction coefficient D versus a/b at different values of ϵ_1' ($\epsilon_2 = 4.75$)

TABLE II
A COMPARISON OF RESULTS USING DIFFERENT THEORIES TO CALCULATE THE EFFECTS OF THE COMPOSITE HOLE FOR SOME WELL-KNOWN MATERIALS

Material	Cavity size (mm)				Calculated ε			Reference Value
	R	R_1	R_2	H	Omit the effects of Hole	Revised by Estin's results [1]	Revised by present theory	
Teflon	51.55	-	3.188	45	2.00	2.05	2.09	2.10 [7]
Quartz	51.55	2.337	3.321	45	3.61	-	3.80	3.78 [7]
Water	51.55	.432	.708	45	74.1	-	77.4	76–78 [8]
Pyrex	45.8	-	12.7	100	4.22	4.39	4.49	4.65*
Nylon	45.8	11.2	12.7	100	2.88	-	3.02	3.02*

* These values are obtained in a closed cavity by Gregory [4] , Fig. 1c.

$C=0.21$). This difference is in part due to the contribution of higher order modes (20 percent to 30 percent of the total value) which were neglected in [1] and [2]. The remaining difference is due to the assumption of the boundary condition at the interface $z'=0$. The first-order perturbation theory leads to a H (6) which will probably be higher than its true value, so our results (13) and (14) will be an upper approximation of the effects of a composite hole.

2) When a dielectric tube is used to introduce the material to be measured (Fig. 1), the effects on the measurement of ϵ^* will be greatly increased with the increased value of ϵ' of the measured material. This situation is of interest on the measurement of biological samples with a high water content.

These theoretical results have already been used to revise experimental data, and the calculated values of the complex dielectric constant of various materials agreed very well with the corresponding values found in the literature [5]. For a brief comparison, some experimental results from the authors and Gregory [4] are treated by both Estin's simple theory [1] and the present method (Table II). Obviously, this more precise calculation is useful for an exact dielectric measurement. In addition, it must emphasize that wall losses of the metallic tube are not considered in this paper. At high frequency, these losses might be an important source of error in dielectric losses measurement.

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Substitution Method for Swept-Frequency Measurements of Dielectric Properties at Microwave Frequencies

MARK J. HAGMANN, MEMBER, IEEE, AND O. P. GANDHI, FELLOW, IEEE

***Abstract*—A solid-state computer-controlled system has been used to make swept-frequency measurements of the insertion loss and reflectance of biological specimens and other media. A substitution procedure was used for direct comparison of samples to allow determination of small differences (on the order of 0.1 dB) in insertion loss and reflectance.**

I. INTRODUCTION

Several investigators claim to have observed sharp and distinct resonances in the absorption of millimeter waves by a number of biochemicals and biological preparations [1]–[5]. Some of the data suggest that the absorption spectra are significantly different for normal and malignant tissues, so that such differences might

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The authors are with the Department of Electrical Engineering, University of Utah, Salt Lake City, UT 84112.

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